Cellular Neural Network Based Platform For Modelling Nonlinear Dynamics of Atrial Fibrillation

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***Abstract*—Atrial Fibrillation (AF) remains the most common clinically encountered arrhythmia (of the heart). Although there are a variety of causes for AF, sudden cardiac death resulting from AF can be separated into two components: initiation of tachycardia and degeneration of tachycardia to fibrillation. There is clinical evidence that the transition from tachycardia to fibrillation is a transition to spatiotemporal chaos, with the underlying dynamics modelled by spiral waves. In this work, we propose a Discrete-Time Cellular Neural/Nonlinear Network (DTCNN) specified using the Open Compute Language (OpenCL) for implementation on a Field-Programmable Gate Array (FPGA). The purpose of this work is to design a robust hardware-software system for implementing a variety of models (Fitzhugh-Nagumo Reaction Diffusion Model, Greenberg-Hastings Cellular Automata) to study the nonlinear (chaotic) dynamics of spiral waves, using the paradigm of DTCNN. The choice of using a DTCNN allows us to incorporate the effects of external inputs, say breathing, to spiral wave dynamics of atrial fibrillation.**

I. Introduction

It is well known that the major event leading to sudden cardiac death is the development of unstable self-sustaining recirculating waves of electrical activity [4]. Despite more than a century of intense research in this field, our understanding of the mechanisms underlying the complex spatio-temporal patterns that result in arrhythmia is incomplete. However, the dynamics of these spatio-temporal patterns can be modelled through the use of spiral waves [4].

The justification for studying wave propagation in the cardiac tissue surface can be understood by the fact that synchronized mechanical activity of the heart depends on the propagation of electrical excitation waves, which trigger the complex sequence of biochemical processes involved in the contraction of individual heart cells [4]. Because each cardiac cell is electrically coupled to its immediate neighbours, the action potential of one cell provides the stimulating current which depolarizes neighbouring cells to trigger their potential [4]. Thus, excitation initiated in one region of the heart forms a wave that propagates rapidly through the entire heart [4]. These waves of excitation are considered nonlinear waves because their shape and amplitude remain constant as they propagate, at the expense of energy taken from the heart cells. It has also been experimentally demonstrated [1], [4], [6], [7], [9], [13] that such nonlinear waves in cardiac tissue fall under the spiral waves category.

Nevertheless, one needs a model that can "sufficiently" reproduce the physical characteristics of spiral waves that are observed in cardiac tissue. Also, we need a robust platform to study the dynamics of spiral waves. The aim of this work is to propose such a platform and utilize the platform to study a variety of models that exhibit spiral waves. Our work also enables one to study the effect of external inputs, such as breathing, on spiral wave dynamics [10]. The underlying "engine" is the DTCNN [2], [3].

This paper is organized as follows: we will first discuss the (DT)CNN, followed by two (due to space limitations) prototypical spiral wave generating models [1], [5] that can be emulated by our hardware platform. We will also briefly justify why we chose these particular models, out of the myriad of models that are available for AF[1]. We next briefly discuss the inexpensive FPGA hardware platform. This is followed by a brief discussion of OpenCL [15]. Using this high level paradigm for specifying DTCNN models enables the software to infer the appropriate data-level parallelism for FPGA implementation. The penultimate section shows DTCNN simulation results from one (due to space limitations) of the models. The paper concludes with a discussion of future work.

II. Mathematical Models

In this section, we will first discuss the general concepts behind the (DT)CNN. Next, we discuss two prototypical models that generate spiral waves. When choosing a mathematical model for a physiological system, the ultimate goal is to have a model that can reproduce physical behaviour, despite the model's simplicity. Based on this criterion, we have chosen two models for studying the dynamics of AF. The Fitzhugh-Nagumo (FN) model is chosen because it can exhibit three characteristics necessary for spiral wave generation: excitability, stimuli, threshold and

[1]After reading our paper, one can infer that our platform is capable of emulating other nonlinear spiral wave generating models, configured as DTCNN

refractiveness [3], [13]. The Greenberg-Hastings Cellular Automata (GHCA) model [1], [5] can help us study global dynamics of spiral waves utilizing only local rules.

A. (DT)CNN

We will first specify the general definitions underlying the standard CNN and then derive the specifics of the DTCNN [2].

Definition 1: A *standard CNN architecture* consists of an $M \times N$ rectangular array of cells $(C(i,j))$ with Cartesian coordiantes (i,j), $i = 1,2,\ldots,M, j = 1,2,\ldots,N$, shown in Fig. 1.

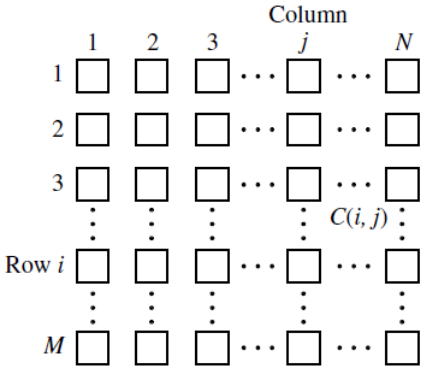


Fig. 1. $M \times N$ CNN architecture.

Definition 2: The *sphere of influence*, $S_r(i,j)$, of the radius r of cell $C(i,j)$ is defined to be the set of all the neighborhood cells satisfying the following property

$$S_r(i,j) = \{C(k,l)|_{\max_{1\le k\le M, 1\le l\le N}}\{|k-i|,|l-j|\} \le r\} \quad (1)$$

where r is a positive integer. We will use the $r = 1$ neighbourhood shown in Fig. 2.

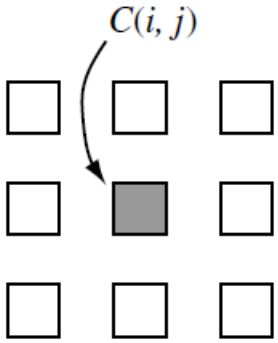


Fig. 2. Definition of $r = 1$ neighbourhood.

Definition 3: A cell $C(i,j)$ is called a *regular cell* with respect to $S_r(i,j)$ iff all neighbourhood cells $C(k,l) \in S_r(i,j)$ exist. Otherwise, $C(i,j)$ is a *boundary cell.*

Definition 4: A *class* $M \times N$ *standard CNN* is defined by an $M \times N$ rectangular array of cells $C(i,j)$ located at site $(i,j), i = 1,2,\ldots,M, j = 1,2,\ldots,N$. Each cell $C(i,j)$ is defined mathematically by:

$$\dot{x}_{ij} = -x_{ij} + \sum_{C(k,l)\in S_r(i,j)} A(i,j;k,l)y_{kl} + \sum_{C(k,l)\in S_r(i,j)} B(i,j;k,l)u_{kl} + z_{ij} \quad (2)$$

where $x_{ij} \in R$, $y_{kl} \in R$, $u_{kl} \in R$ and $z_{ij} \in R$ are called the state, output, input and threshold of cell $C(i,j)$ respectively. $A(i,j;k,l)$ and $B(i,j;k,l)$ are called the feedback and the input synaptic operators respectively. In this paper, we will assume that both operators are space and time invariant. Eq.(2) is the *state equation.* Eq.(3) is the *output equation* with the standard nonlinearity.

$$y_{ij} = \frac{1}{2}\left(|x_{ij}+1| - |x_{ij}-1|\right) \quad (3)$$

We also neeed to specify *boundary conditions* for Eq.(2). These conditions specify y_{kl} and u_{kl} for cells belonging to $S_r(i,j)$ of the outermost boundary cells (edge cells) and these edge cells lie outside the $M \times N$ array. Finally, we can also specify *initial conditions*, as shown in Eq.(4).

$$x_{ij}(0), \qquad i = 1,\ldots,M, \qquad j = 1,\ldots,N \quad (4)$$

Since our hardware platform (FPGA) is a sampled and quantized system. Therefore, we can apply Euler's method to Eqs.(2) and (3) to get Eq.(5).

$$\begin{aligned} x_{ij}((m+1)\Delta t) &\triangleq x_{ij}(m+1) \\ &= (1-\Delta t)x_{ij}(m) \\ &\quad + \Delta t \sum_{k=-r}^{r}\sum_{l=-r}^{r} a_{kl}y_{i+k,j+l}(m) \\ &\quad + \Delta t \sum_{k=-r}^{r}\sum_{l=-r}^{r} b_{kl}u_{i+k,j+l}(m) + \Delta t \cdot z \\ y(x_{kl}(m+1)) &= f(x_{kl}(m+1)) \end{aligned} \quad (5)$$

where Δt is defined in normalized time units. f is the nonlinear function defined in Eq.(3). Note that we have assumed our operators are both space and time-invariant.

If we use $\Delta t = 1$ in Eq.(5), we obtain the standard DTCNN. Due to the high clock frequency of 21st century FPGAs, we can reduce Δt, if there are convergence issues with the DTCNN.

We will now discuss two models of atrial fibrillation. Both models can be specified in terms of a DTCNN, but due to space limitations we will focus on the FN model [3].

B. FN Model

Consider the FN model with unidirectional diffusion in Eqs.(6) and (7).

$$\frac{\partial u}{\partial t} = f(u,v) + D\nabla^2 u \quad (6)$$

$$\frac{\partial v}{\partial t} = g(u,v) \quad (7)$$

The FN is a reaction-diffusion model because it consists of kinetic terms, f and g, and a diffusion term, $D\nabla^2 u$ [13]. The variable u is an activator, representing a normalized membrane potential and the variable v is an inhibitor, representing the compound effect of all deactivating and inhibiting mechanisms of the cell membrane. Due to gap junctions the potential u is diffusive but v is not since the membrane is practically impermeable to ions. The nonlinear functions f and g are shown in Eqs.(8) and (9) [3].

$$f(u,v) = cu - \frac{u^3}{3} - v \tag{8}$$

$$g(u,v) = -\epsilon\,(u - bv + a) \tag{9}$$

We can discretize Eqs.(6) through (9) by introducing "discretized" spatial coordinates in the $x-y$ plane and by a "discretized" two-dimensional Laplacian [3] with a diffusion coefficient D. Hence the CNN model that we will implement on the FPGA is shown in Eqs.(10), (11) and (12).

$$\frac{du(j,k)}{dt} = f(u(j,k),v(j,k)) + I_1(j,k) \tag{10}$$

$$\frac{dv(j,k)}{dt} = g(u(j,k),v(j,k)) \tag{11}$$

$$\begin{aligned} I_1(j,k) = D(&u(j-1,k) + u(j+1,k) \\ &+ u(j,k-1) + u(j,k+1) - 4u(j,k)) \end{aligned} \tag{12}$$

C. GHCA Model

The GHCA model was chosen because it captures the effects of a heterogeneous cellular structure using local rules, while providing computational efficiency [1]. The original GHCA formulation is simple: each site at time t is assigned a state, $u_j(t)$, where the subscript j refers to the cell number. The neighbourhood of a given site is determined using a square (Moore) metric of diameter r. The state is an integer: 0 is a rest state; states $1, 2, \ldots, E$ are excited states; states $E+1, E+2, \ldots, E+R$ are refractory states; state $(E+R+1)$ is identified with rest state 0. The update rule for the state of a site is: If $(1 \leq u_i(t) \leq E+R)$, then $u_i(t+1) = u_i(t) + 1$; and if $(u_i(t) = 0)$, then $u_i(t+1) = 0$ unless u_i is excited by its neighbours, or becomes spontaneously active with probability p_s.

Note that we need to introduce spatial heterogenity and assign random weights for a cell's influence on its neighbours, in order for the wave propagation in GHCA to closely mimic atrial fibrillation dynamics [1].

Now that we have discussed two viable mathematical models, we will next illustrate the hardware platform that is inexpensive, yet can implement mathematical models of atrial fibrillation.

III. Hardware Platform

For this work, we use the DE1-SOC from Terasic Inc., at a cost of only US$ 150 (academic) per board [14]. This board has the Cyclone V [11] FPGA that incorporates a dual-core ARM controller along with the FPGA fabric. Hence the ARM processor is utilized for executing sequential control code while the FPGA fabric implements the data-driven parallelism required by the DTCNN. A block diagram of our system is shown in Fig. 3.

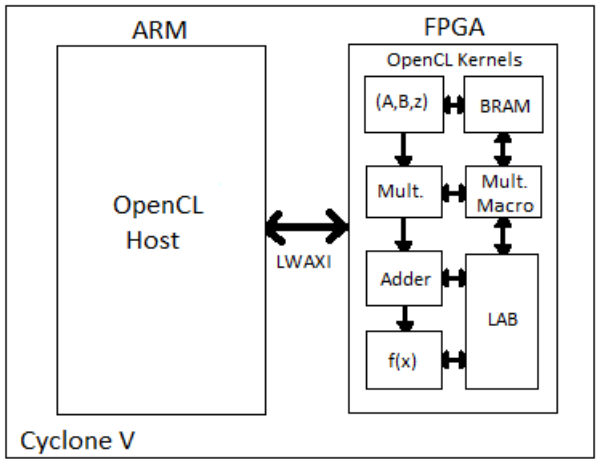


Fig. 3. A high level block diagram of our DTCNN implementation on the Cyclone V. The OpenCL kernels on the FPGA are executed in parallel over an r neighourhood. The host process on the ARM core(s) coordinates kernel execution on the FPGA via the LWAXI (LightWeight Advanced eXtensible Interface). Not shown in the block diagram are external peripheral interfaces (such as VGA, for visualizing spiral wave dynamics in Fig. 4).

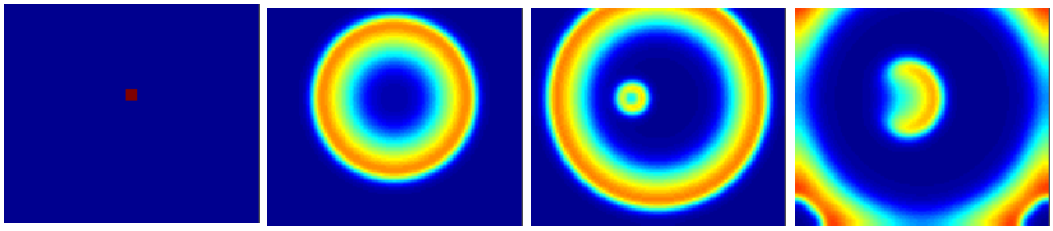
IV. Software Paradigm

The spatial-convolution operator in a DTCNN can be implemented in a sequential programming language using nested loops. But to take advantage of FPGA parallelism, we use the OpenCL C language specification [15] to map the nested loops as a two-dimensional operation over the DTCNN r neighbourhood.

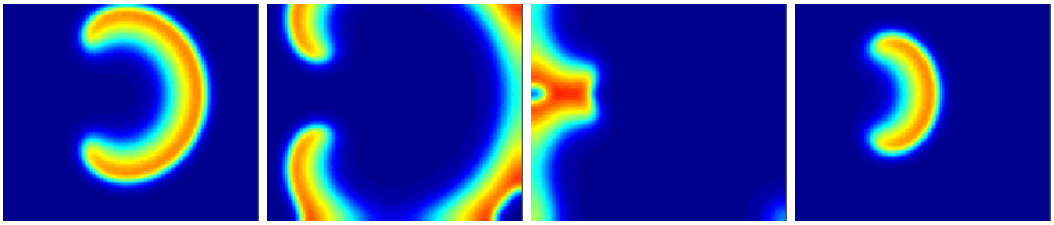
Simply put, we have a OpenCL process running on the ARM controller (control node or "host") that coordinates parallel computation of the input convolution, feedback convolution and threshold operation on the FPGA. Each cell $(C_r(i,j))$ in the r neighbourhood acts as a device ("core") in OpenCL terminology. Once all computations over the r neighbourhood are accomplished in parallel, the process is sequentially repeated over the entire image to complete one iteration of the DTCNN[2].

In the penultimate section, we will show spiral wave dynamics only from the FN model (due to space limitations). Nevertheless, we plan to provide a demonstration of our physical platform incorporating different models [7] at ISCAS 2015, because of hardware portability.

[2]The restriction to an r neighbourhood is because of onchip FPGA memory limitations on our inexpensive platform.



(a) $t = 0$ (b) $t = 7$ (c) $t = 9$ (d) $t = 13$



(e) $t = 16$ (f) $t = 21$ (g) $t = 23$ (h) $t = 30$

Fig. 4. Spiral wave results from DTCNN specification of FN model. We have used a 256×256 CNN and the VGA output is appropriately scaled. One can infer the onset of spiral wave activity at $t = 13$. In the given window, the two foci of the spiral wave collide and the pattern from $t = 13$ repeats at $t \approx 30$.

V. Spiral Wave Generation: Results

To generate spiral waves, we implemented Eqs.(10) through (12) with the following parameter values: $a = 0.1, b = 1.4, c = 1, \epsilon = -0.1, D = 0.1$ [3]. Results are shown in Fig. 4.

VI. Conclusion

In this paper we have illustrated a robust hardware-software platofrm for implementing a variety of DTCNN models that are capable of generating spiral waves. The goal of this work is for our platform to serve as a 21st century enabling technology that utilizes CNN for the application domain of studying human arrhythmia dynamics, and its subsequent control.

A starting point for futre work would be to understand the role played by chaotic dynamics in atrial fibrillation, by considering the effect of an external input, particularly breathing [10]. It would also be insightful to compare the results from our platform to results from a general purpose multi-scale simulation package, such as CHASTE [12].

With respect to our research into atrial fibrillation, we are planning on utilizing the FN model. The reason is that this model can display biophysically meaningful chaotic spatio-temporal patterns [3], despite its simplicity.

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References

[1] G. Bub., A. Shrier and L. Glass. "Bursting in Cellular Automata and Cardiac Arrhythmias", *Chaos, CNN, Memristors and Beyond*. World Scientific, 2013.

[2] L. O. Chua and T. Roska, "Cellular Neural Networks and Visual Computing: Foundations and Applications", Cambridge University Press, 2002.

[3] R. Dogaru and L. O. Chua, "Edge of Chaos and Local Activity Domain of FitzHugh-Nagumo Equation", *International Journal of Bifurcation and Chaos*, **08(2)**, pp. 211-257, 1998.

[4] R. A. Gray and J. Jalife, "Spiral Waves and the Heart", *International Journal of Bifurcation and Chaos*, **6(3)**, pp. 415-435, 1996.

[5] J. M. Greenberg and S. P. Hastings, "Spatial Patterns for Discrete Models of Diffusion in Excitable Media", *SIAM Journal of Applied Mathematics*,**34(3)**, pp. 515-523, 1978.

[6] S. T. Mathew., J. Patel and S. Joseph, "Atrial Fibrillation: Mechanistic Insights and Treatment Options", *European Journal of Internal Medicine*, **20(7)**, pp. 672-681, 2009.

[7] W-J. Rappel and S. M. Narayan, "Theoretical Considerations for Mapping Activation in Human Cardiac Fibrillation", *Chaos*, **23**, p. 023113, 2013.

[8] C. K. Subramaniam et. al., "CNN for Identification of Retinal Defects", *Submitted for IEEE ISCAS 2015 Review.*

[9] J. N. Weiss et. al., "Chaos and the Transition to Ventricular Fibrillation : A New Approach to Antiarrhythmic Drug Evaluation", *Circulation*, **99**, pp. 2819-2826, 1999.

[10] N. Wessel., M. Riedl and J. Kurths, "Is the Normal Heart Rate "Chaotic" Due to Respiration?", *Chaos*, **19**, p. 028508, 2009.

[11] Altera Corporation. "Documentation: Cyclone V Devices", Available online: http://www.altera.com/literature/lit-cyclone-v.jsp Last accessed: 5/11/2014

[12] University of Oxford. "CHASTE Simulation Package", Available online: http://www.cs.ox.ac.uk/projects/chaste/ Last accessed: 5/11/2014

[13] M. Phillip. "Mathematical Modelling of Cardiac Arrhythmias", Available online: http://www.student.nada.kth.se/~f97~mph/rapport/rapport.html Last accessed: 4/11/2014

[14] Terasic Inc. "DE1-SOC", Available online: http://de1-soc.terasic.com Last accessed: 5/11/2014

[15] R. Tsuchiyama et. al. "The OpenCL Programming Book", Available online: http://www.fixstars.com/en/opencl/book/ Last accessed: 5/11/2014