Two Element Chaotic and Hyperchaotic Circuits

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Goal(s) of This Work

1. Design a robust and easy to use digital platform for physical emulation of circuit elements with memory

2. Illustrate robustness and applications of our emulator by realizing two element chaotic and hyperchaotic circuits.

Outline

I. Prerequisites for understanding this talk:
  1. First course in circuit theory
  2. First course in differential equations

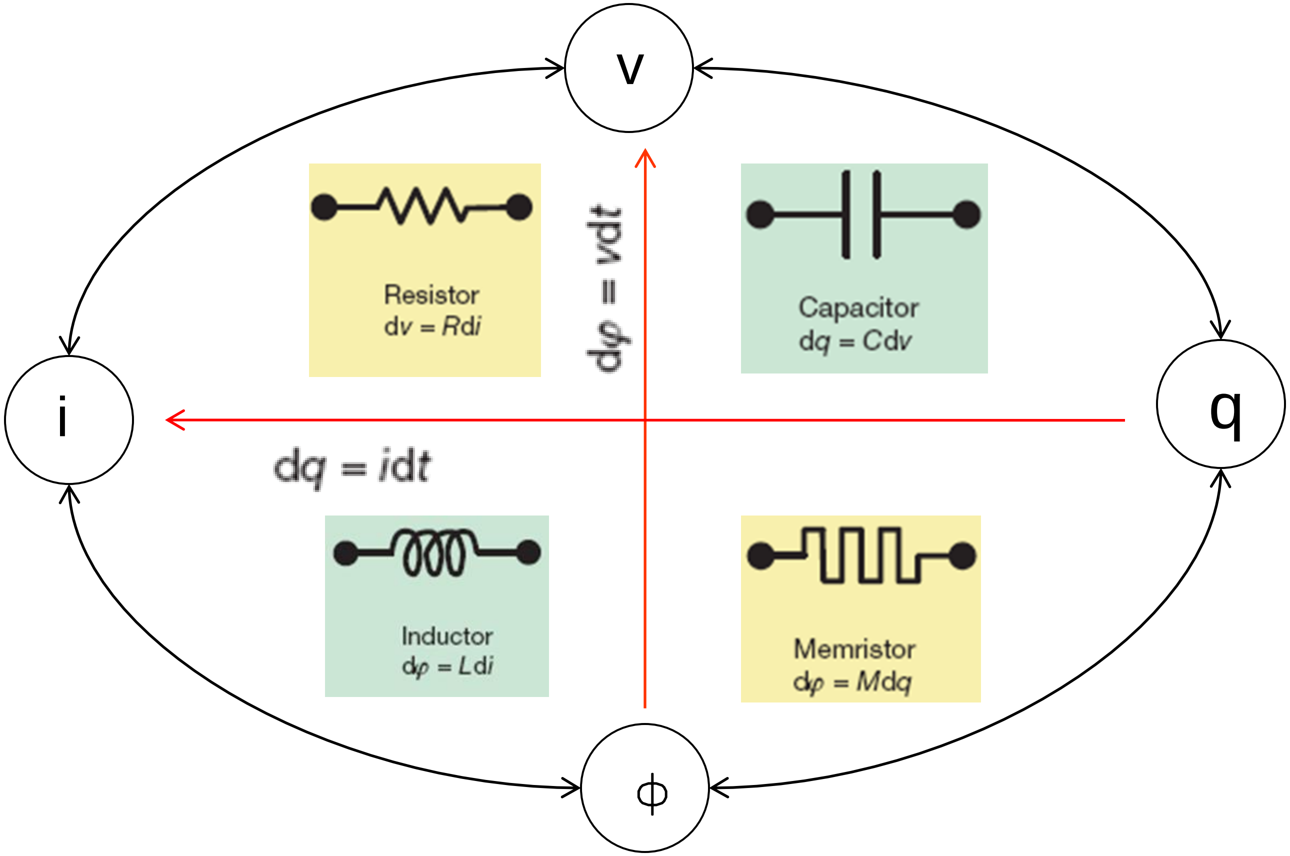
II. Background
  1. The Fundamental Circuit Elements
  2. Circuit Elements with Memory
  2. An Introduction to Chaos (and Hyperchaos)

III. The Emulator
  1. Block Diagram of our System
  2. Pinched Hysteresis Loop

IV. Two Element Chaotic Circuit Example
  1. Circuit Equations
  2. Simulation Results

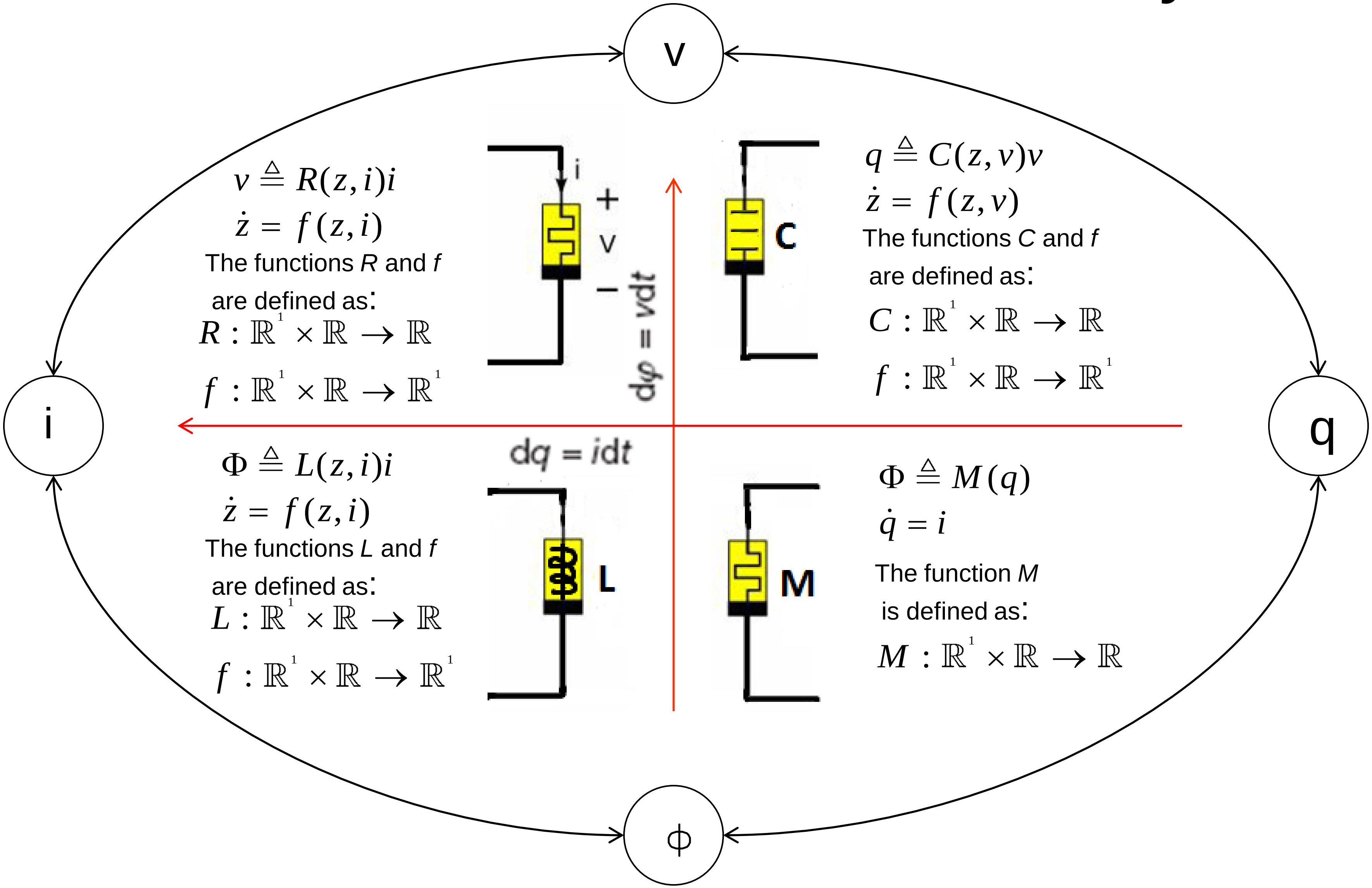
V. Conclusions, Future work and References

The Fundamental Circuit Elements



Memristors were first postulated by Leon. O Chua in 1971 [2], realized by Strukov et. al. [13]

Circuit Elements with Memory



$$v \triangleq R(z,i)i$$
$$\dot{z} = f(z,i)$$

The functions *R* and *f* are defined as:

$$R : \mathbb{R}^1 \times \mathbb{R} \to \mathbb{R}$$
$$f : \mathbb{R}^1 \times \mathbb{R} \to \mathbb{R}^1$$

$$q \triangleq C(z,v)v$$
$$\dot{z} = f(z,v)$$

The functions *C* and *f* are defined as:

$$C : \mathbb{R}^1 \times \mathbb{R} \to \mathbb{R}$$
$$f : \mathbb{R}^1 \times \mathbb{R} \to \mathbb{R}^1$$

$$\Phi \triangleq L(z,i)i$$
$$\dot{z} = f(z,i)$$

The functions *L* and *f* are defined as:

$$L : \mathbb{R}^1 \times \mathbb{R} \to \mathbb{R}$$
$$f : \mathbb{R}^1 \times \mathbb{R} \to \mathbb{R}^1$$

$$\Phi \triangleq M(q)$$
$$\dot{q} = i$$

The function *M* is defined as:

$$M : \mathbb{R}^1 \times \mathbb{R} \to \mathbb{R}$$

An Introduction to Chaos (and Hyperchaos)

- "Birth" of Chaos: Lorenz Attractor [1] [10]
  - Edward Lorenz introduced the following nonlinear system of differential equations as a crude model of weather in 1963:

$$\dot{x} = -\sigma \cdot x + \sigma \cdot y$$

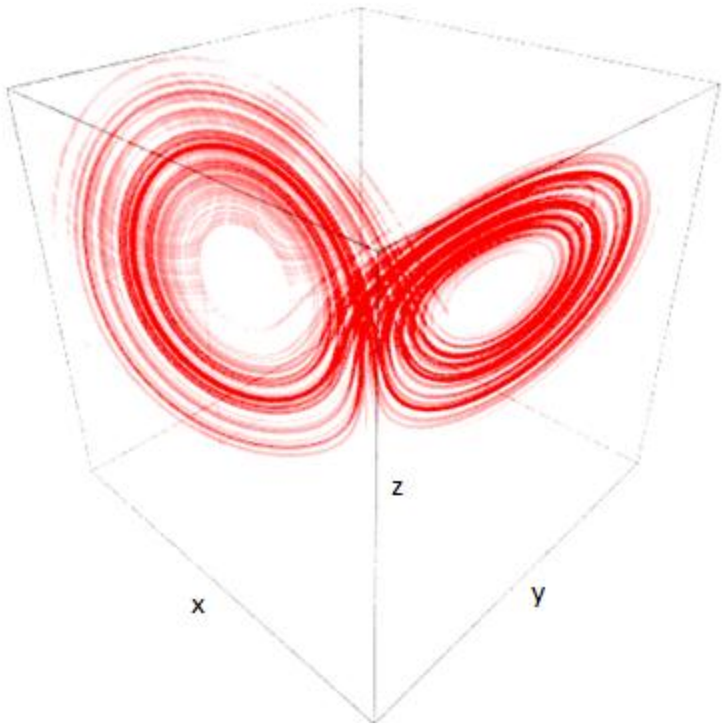
$$\dot{y} = \rho \cdot x - y - x \cdot z$$

$$\dot{z} = -\beta \cdot z + x \cdot y$$

$$\text{Parameters:} \sigma = 10, \rho = 28, \beta = \frac{8}{3}$$

$$\text{ICs}: x_0 = 10, y_0 = 20, z_0 = 30,$$

Simulation time: 100 seconds



Note : Image rendered in SAGE using tachyon ray tracing engine

  - Lorenz discovered that model dynamics were extremely sensitive to initial conditions and the trajectories were aperiodic but bounded.
  - But, does chaos exist *physically?* Answer is: YES. For example, Chua's circuit [4].
  - Hyperchaos is higher dimensional chaos.

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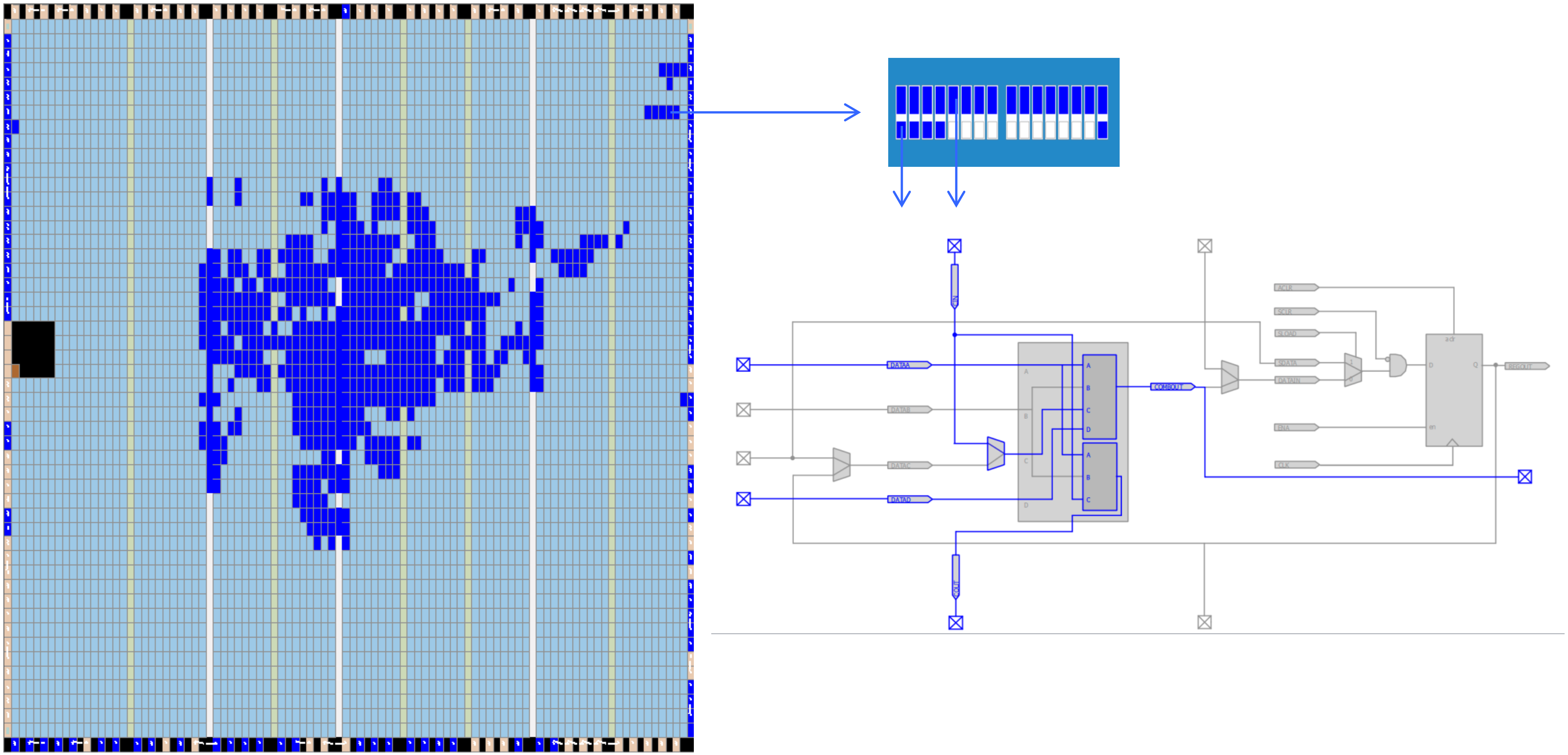
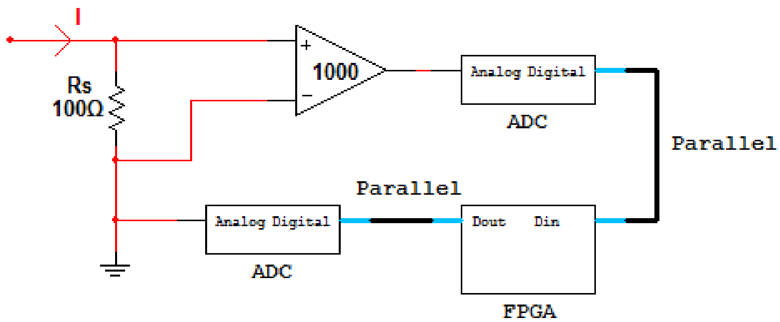
Block Diagram of our System – What is an FPGA?

FPGA is an acronym, Field Programmable Gate Array

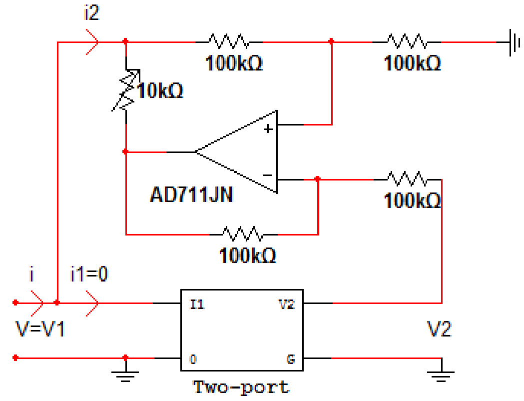


Block Diagram of our System (contd.)

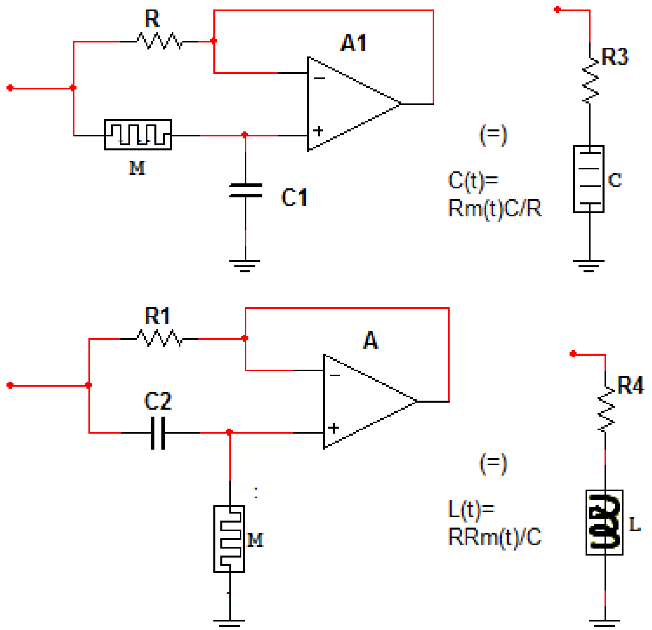
Current-controlled realization



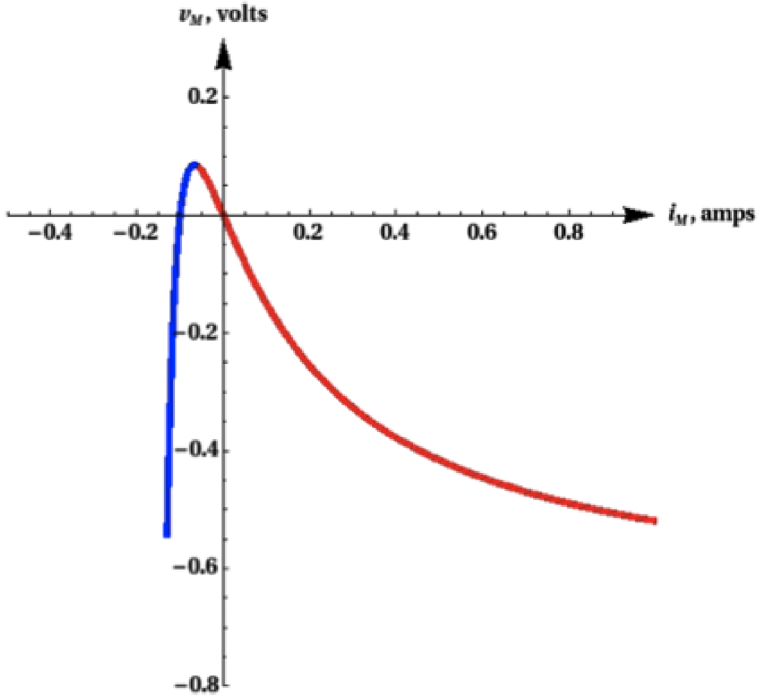
Voltage-controlled realization [5]

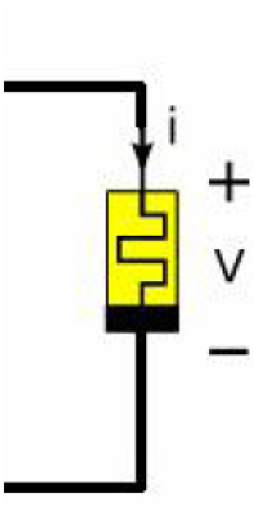


Memcapacitor-meminductor converter [11]



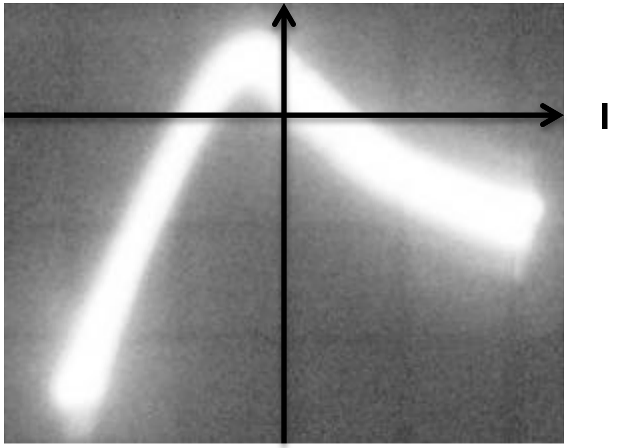
DC v-i and Pinched Hysteresis Loop

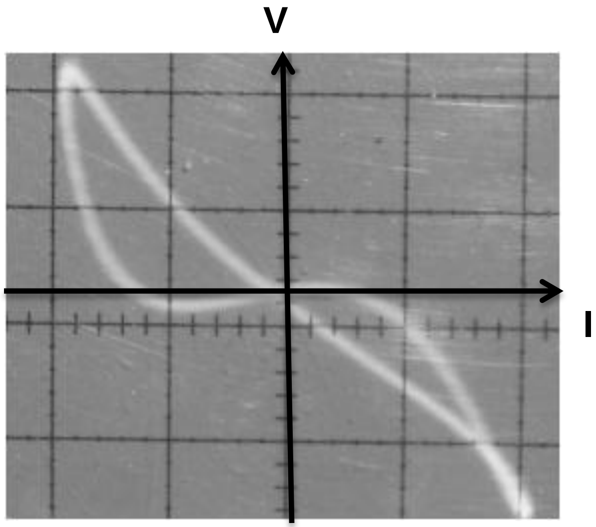


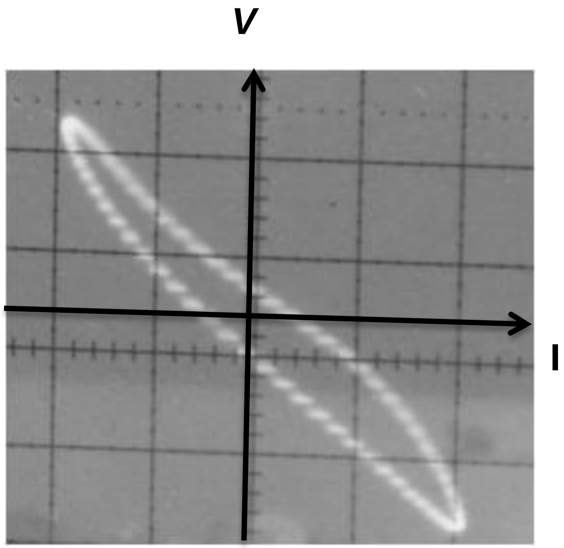


$$v \triangleq R(z,i)\cdot i$$

$$\dot{z} = f(z,i)$$







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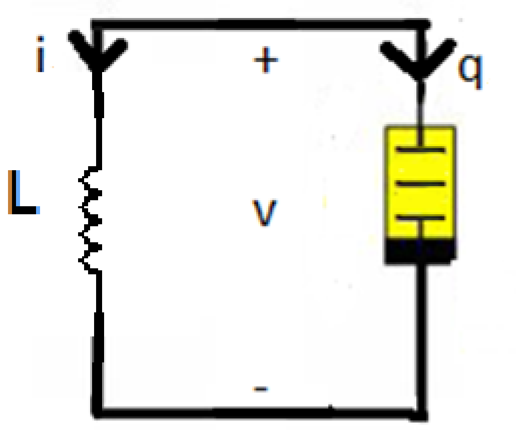
LM_C circuit – equations (dimensionless)



Fully nonlinear memcapacitve system characteristics:

$$\dot{x}_M \equiv -v - \alpha x_M + v \cdot x_M$$

$$q = Cv + q_M(x_M) = Cv + \beta\left(\frac{x_M^3}{2} - x_M\right)$$

Circuit Equations:

$$\dot{x}_M = -v - \alpha x_M + v \cdot x_M$$

$$\dot{v} = \frac{-1}{C}\left(i + \beta(x_M^2 - 1)v\right)$$

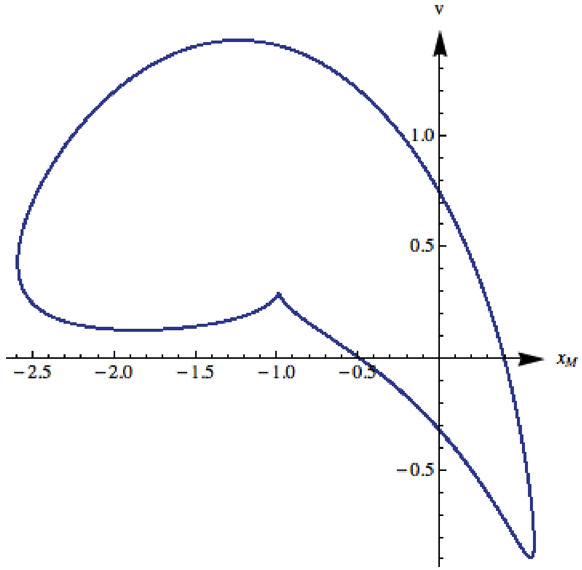
$$i' = \frac{v}{L}$$

Parameters and initial conditions (for chaos):

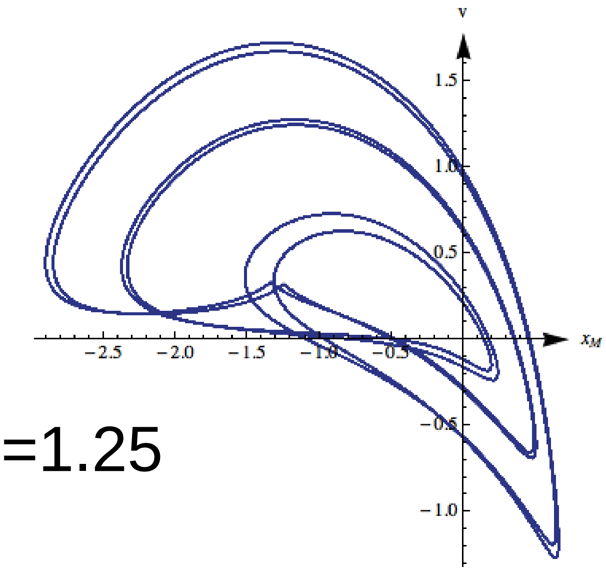
$$\alpha = 0.6, \beta = 1.5, C = 3, L = 1$$

$$x(0) = 0.1, v(0) = 0, i(0) = 0.1$$

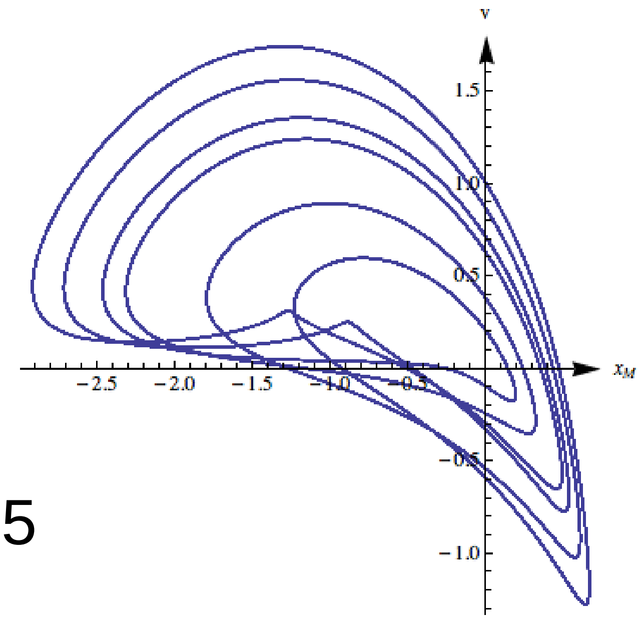
LM_C circuit – simulation results



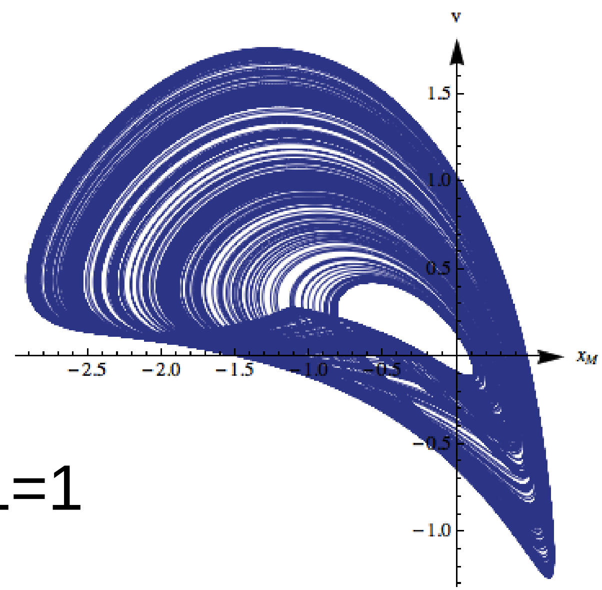
L=1.5



L=1.25



L=1.15



L=1

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Conclusions and Future Work

1. We illustrated use of FPGAs for emulating circuit elements with memory (nonlinear dynamical systems)

2. Future work :
   1. Complete physical realization of two element chaotic and hyperchaotic circuits (mid-May 2012)
   2. Jitter analysis (end of 2012)

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Discussion